

Meshless methods for PDEs on manifolds

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We consider a plane waves method as a numerical technique for solving boundary value problems for linear partial differential equations on manifolds (see [1]). Like other meshless methods [2], this may also be considered for specific interpolation purposes. The plane waves method is here applied to Laplace-Beltrami and Helmholtz-Beltrami equations. Density results justify the completeness of the plane waves space as an approximation for domain and boundary data. A-posteriori error estimates and numerical experiments show that this simple technique may be used to accurately solve boundary value problems on manifolds. In the next figure we present a numerical result for a cylindrical surface domain, where the top and bottom circles are the two parts of the boundary.

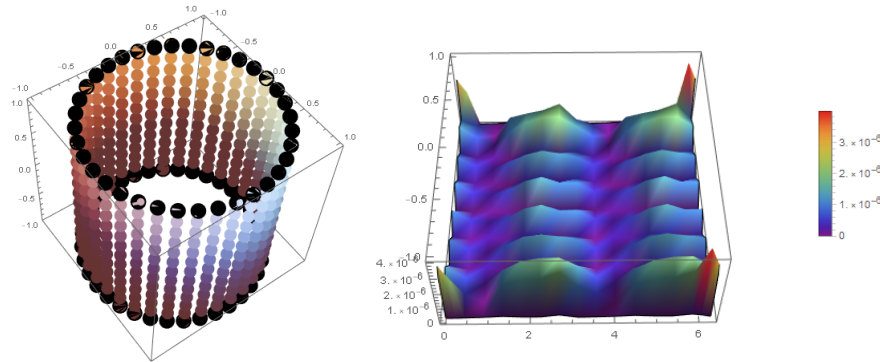


Figure: On the left, the geometry of the domain (a surface of a cylinder) with the collocation points (here black points are on the boundary). On the right, the residual error plot in the domain is $\approx 10^{-6}$ (while the boundary error for the Dirichlet problem is $\approx 10^{-8}$).

- [1] Alves CJS, Antunes PRS, Martins NFM, Valtchev SSV (2020) Solving boundary value problems on manifolds with a plane waves method. *Applied Mathematics Letters*: 107, 106426.
- [2] Chen M, Cheung KC, Ling L (2020) Solving Partial Differential Equations on Surfaces with Fundamental Solutions; in *Advances in Trefftz Methods and Their Applications* (edited by C. Alves, A. Karageorghis, V. Leitão, S. Valtchev), SEMA SIMAI Springer Series (pp. 1-11).