Counting curves in space: maps or equations?

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Counting the numbers of curves in space satisfying certain conditions (for example meeting certain points, having certain degree or genus, etc.) has been a central topic in algebraic geometry since the 19th century, and goes back even to ancient Greece.

The last 30 years have seen a great development of these ideas, allowing the formulation of "curve counts" in very general situations through the theory of moduli spaces and virtual integration. One of the aspects of the modern theory is that there are two fundamentally different ways to think of curve counts: we either think of curves on a space X as a map from the curve to X (leading to Gromov–Witten theory), or we think of the equations that define the curve in X (Donaldson–Thomas and Pandharipande–Thomas theories). Conjecturally, these different perspectives actually produce "equivalent" numbers. In some sense, the "equations" side is simpler and has helped proving results on the more classical "maps" side; in the talk I'll illustrate this principle with a couple of recent examples.

Invited Algebraic Geometry Session