

Main speaker (45 minutes talk)

## Modulated Bi-orthogonal Polynomials on the Unit Circle: The $2j - k$ and $j - 2k$ Systems

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We construct the systems of bi-orthogonal polynomials on the unit circle where the Toeplitz structure of the moment determinants is replaced by  $\det(w_{2j-k})_{0 \leq j, k \leq N-1}$  and the corresponding Vandermonde modulus squared is replaced by  $\prod_{1 \leq j < k \leq N} (\zeta_k^2 - \zeta_j^2)(\zeta_k^{-1} - \zeta_j^{-1})$ . This is the simplest case of a general system with structure  $pj - qk$  where  $p, q$  are co-prime integers. We derive analogues of the structures well known in the Toeplitz case: third order recurrence relations, determinantal and multiple-integral representations, their reproducing kernel and Christoffel-Darboux sum, and associated (Carathéodory) functions. We give full explicit details for the system defined by the simple weight  $w(\zeta) = e^\zeta$ , which is a specialisation of a weight arising from averages of moments of derivatives of characteristic polynomials over the classical groups  $\text{USp}(2N)$ ,  $\text{SO}(2N)$  and  $\text{O}^-(2N)$ .

(Joint work with Roozbeh Gharakhloo, Colorado State University)

**Submission for an invited talk (main speaker) in the parallel session "Random matrices, integrable systems and orthogonal polynomials" (organized by Maria das Neves Rebocho - UBI, Portugal; Rostyslav Kozhan - Uppsala University, Sweden.)**