Double periodic viscous flows in infinite space-periodic pipes

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We study the motion of an incompressible fluid in an $n+1$-dimensional infinite pipe Λ with an L-periodic shape in the z-axial direction. Below $x=(x_1, x_2, \cdots, x_n)$, and $z=x_{n+1}$. Fluid motion is described here by the evolution Stokes or Navier-Stokes equations together with the non-slip boundary condition $\mathbf{v}=0$. Let $g(t)$ be a given real T-periodic function. We look for solutions $\mathbf{v}(x,z,t)$ with periodic total flux $\int_{\Sigma_z} v_z \, d\Sigma_z = g(t)$, where Σ_z denotes the cross section of the pipe at the level z. We look for solutions which are simultaneously T-periodic with respect to time and L-periodic with respect to z, for $z \in \mathbb{R}$ and $t \in \mathbb{R}$. We prove existence and uniqueness of the solution to the above problem. Furthermore, we show that in the significant case of a $3-D$ symmetrical rotation pipe, $\mathbf{v}$ is just the *full-developed* solution.

To avoid hiding our main ideas under technical calculations, the argument is presented through a sequence of steps. We start by considering the linear, stationary, Stokes problem. Then, by taking this case as a reference, we study the evolution Stokes equations. Finally, we end with the extension to the full Navier-Stokes equations.

References

[1] H. Beirão da Veiga and J. Yang, *Double periodic viscous flows in infinite space-periodic pipes*, arXiv:2103.05913.